P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at 'A+' level by NAAC
Autonomous & ISO 9001:2015 Certified

Title of the Course: ALGEBRA
Semester : I

Course Code	23MA1T3	Course Delivery Method	Blended Mode
Credits	5	CIA Marks	30
No. of Lecture Hours / Week	5	Semester End Exam Marks	70
Total Number of Lecture Hours	75	Total Marks	100
Year of Introduction : 2020-2021	Year of offering : 2023-2024	Year of Revision: 2023-24	Percentage of Revision :5%

Course Objectives: The main objective of this course is to acquire knowledge on the basic concepts of Group theory and Ring theory.

Course Outcomes: After successful completion of this course, students will be able to

CO1: understand the properties of Groups and homomorphisms. (PO1)

CO2: study permutation groups and Cayley's theorem.(PO3)

CO3: study the applications of Sylow's theorems.(PO3)

CO4: understand the properties of ideals in rings, Quotient rings, integral domains and fields. (PO3)

CO5: illustrate the properties of Euclidean rings and polynomial rings.(PO1)

UNIT-I

Group Theory: Definition of a Group, Some Examples of Groups, Some Preliminary Lemmas, Subgroups, A Counting Principle, Normal Subgroups and Quotient Groups, Homomorphisms, Automorphisms. [Sections 2.1 to 2.8 of the prescribed book]

UNIT-II

Group Theory Continued: Cayley's theorem, Permutation groups, Another counting principle. [Sections 2.9 to 2.11 of the prescribed book]

UNIT-III

Group Theory Continued: Sylow's theorem, direct products, finite abelian groups.

[Sections 2.12 to 2.14 of the prescribed book]

UNIT-IV

Ring Theory: Definition and Examples of Rings, Some special classes of Rings, Homomorphisms, Ideals and quotient Rings, More Ideals and quotient Rings, The field of quotients of an Integral domain. [Sections 3.1 to 3.6 of the prescribed book]

UNIT-V

Ring Theory Continued: Euclidean rings, A particular Euclidean ring, Polynomial Rings, Polynomials over the rational field, Polynomial Rings over Commutative Rings.

[Sections 3.7 to 3.11 of the Prescribed book].

PRESCRIBED BOOK:

1. I.N. Herstein, **Topics in Algebra,** Second Edition, Wiley Eastern Limited, New Delhi, 1988.

REFERENCE BOOKS:

1. Bhattacharya P.B., Jain S.K., Nagpaul S.R., **"Basic Abstract Algebra",** Second Edition, Cambridge Press.
2. David S Dummit and Richard M Foote, **“Abstract Algebra”**, Wiley Publications, Third Edition.
3. C. Musili, **“Introduction to Rings and Modules”,** Narosa Publications.

Course has Focus on : Foundation

Websites of Interest: 1. www. nptel.ac.in

2. www.epgp.inflibnet.ac.in
3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An Autonomous college in the jurisdiction of Krishna University)

M. Sc. Mathematics

First Semester

ALGEBRA-23MA1T3

Time: 3 Hours **Max Marks: 70**

SECTION - A

Answer all questions **(5X4 = 20)**

1 (a) If G is a finite group and a ϵ G then prove that $a^{o(G)} = e$. (CO1, L2)

(OR)

(b) Define a subgroup and a normal subgroup. (CO1, L2)

2 (a) If G is a group of order 36 and H is a subgroup of order 9, prove that G cannot be simple. (CO2, L3)

(OR)

(b) Find the product of the permutations $\Phi = (1\ 2\ 3\ 4)$ and $\psi = (3\ 4\ 5\ 1)$ (CO2, L3)

3 (a) Define a p-sylow subgroup and give an example. (CO3, L1)

(OR)

(b) Define external direct product and internal direct product of groups. (CO3, L1)

4 (a) Prove that a finite integral domain is a field. (CO4, L2)

(OR)

(b) Define a homomorphism of rings and give an example. (CO4, L2)

5 (a) Define Euclidean Ring and give an example. (CO5, L1)

(OR)

(b) Define an irreducible polynomial and a primitive polynomial over a field F. (CO5, L1)

SECTION - B

Answer all questions. All questions carry equal marks. **(5X10 = 50)**

6 (a) If H and K are finite subgroups of G of orders o(H) and o(K) respectively, then show that $o(HK) = o(H)o(K) / o(H \cap K)$. (CO1, L2)

(OR)

(b) State and prove the fundamental theorem of homomorphism in groups. (CO1, L2)

7 (a) Show that every group is isomorphic to a subgroup of A(S), for some appropriate S. (CO2, L2)

(OR)

(b) State and prove Cauchy's theorem. (CO2, L2)

8 (a) Show that any two Sylow subgroups of a group G are conjugate. (CO3, L2)

(OR)

(b) Show that If G and G^1 are isomorphic abelian groups, then show that G(s) and G^1(s) are isomorphic, for every integer s. (CO3, L2)

9 (a) If U is an ideal of a ring R, then show that R/U is a ring and is a homomorphic image of R. (CO4, L2)

(OR)

(b) If R is a commutative ring with unity and M is an ideal of R, then prove that M is maximal if and only if R/M is a field. (CO4, L2)

10(a) Prove that J[i], the ring of Gaussian integers is a Euclidean ring. (CO5, L2)

(OR)

(b) State and prove Gauss Lemma. (CO5, L2)

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